FEM vs. FDM / FVM
Some Essential Differences

$$r c_p \frac{\P T}{\P t} = \frac{\P}{\P x}\left(l \frac{\P T}{\P x}\right) + \frac{\P}{\P y}\left(l \frac{\P T}{\P y}\right) + \frac{\P}{\P z}\left(l \frac{\P T}{\P z}\right) + \&$$

The differential equation of Fourier describing heat conduction. (1)

$$\left(r c_p\right)(T)\left(\frac{T(x,t+\Delta t) - T(x,t)}{\Delta t}\right) =$$

$$l\,(T)\left[\frac{T(x+\Delta x,t) - 2T(x,t) + T(x-\Delta x,t)}{(\Delta x)^2}\right]$$

FDM/FVM:

The differential equation will be transformed into a difference equation (see here the one dimensional case) (2)

$$I(T) := \iiint_V \left\{ \frac{l}{2}\left[\left(\frac{\partial T}{\partial x}\right)^{(2)} + \left(\frac{\partial T}{\partial y}\right)^{(2)} + \left(\frac{\partial T}{\partial z}\right)^{(2)}\right] + r c_p T \frac{\partial T}{\partial t} - \dot{Q} T \right\} dxdydz$$

$$+ \iint_R \left\{ \frac{1}{2} a(u,v) T^2 - g(u,v) T \right\} dudv = !\min imal$$

FEM:

The differential equation will be solved with an equivalent variation problem (3)

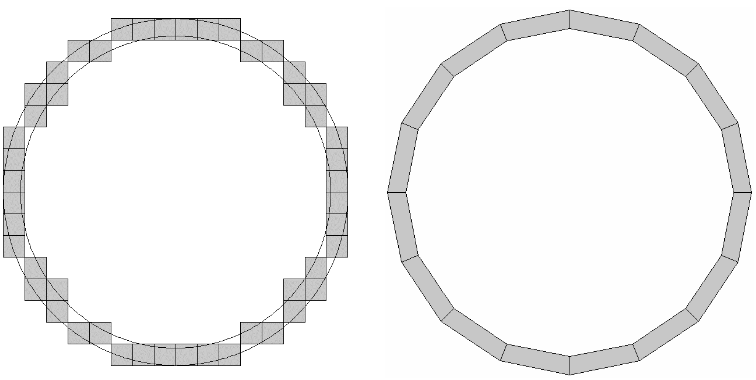

Figure 1:

FDM/FVM:	**48 nodes** **108 elements**	**FEM:**	**32 nodes** **16 elements**

To solve a differential equation with the help of the finite difference method (FDM) or the finite volume method (FVM) the differential equation will simply be changed into a difference equation (2). To solve the difference equation the geometry will be divided into an orthogonal grid. As an example of heat conduction we are using one equation and one unknown value for all squares.

Using the finite element method (FEM) the differential equation will be solved by solving an equivalent variation problem. Even here the geometry must be divided into small elements, but while the FDM/FVM needs a rectangular grid, FEM can handle nearly all basic elements. The variation problem will be solved by assuming a particular function of the unknown value. For example the assumption is a linear or quadratic function. The variation problem can be integrated by using this function. Even if the user of FDM/FVM programs can not see it at times, the geometry approximation is poor compared to an enmeshment with Finite Elements. This is not just an optical problem. In the example of a cylinder with constant wall thickness, it is not possible to enmesh such a cylinder with a rectangular grid while keeping a constant wall thickness (fig.: 1). This means that calculating the solidification of a cylinder leads to a different temperature distribution along the circumference. But the calculation is so inaccurate that the different wall thickness doesn't matter.

The disadvantage of the rectangular grid will even be more obvious when dealing with heat transfer problems.

The heat transfer can be calculated by:

$$\dot{Q} = a\ \mathbf{A}\ D\mathbf{T}$$

with A as relevant surface.

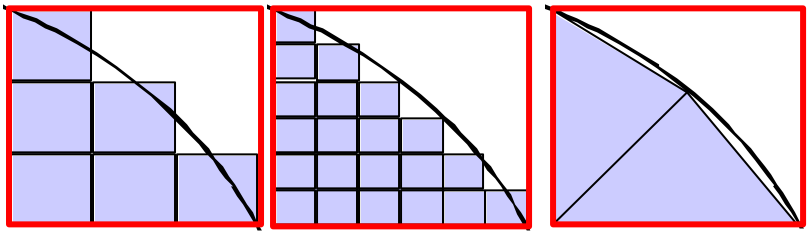

Figure 2:

FDM/FVM:

Even with very small elements, it isn't possible to change the inaccurate factor of √2 between real surface and surface of the mesh

FEM:

Even with few elements the difference between real surface and enmeshed surface is small

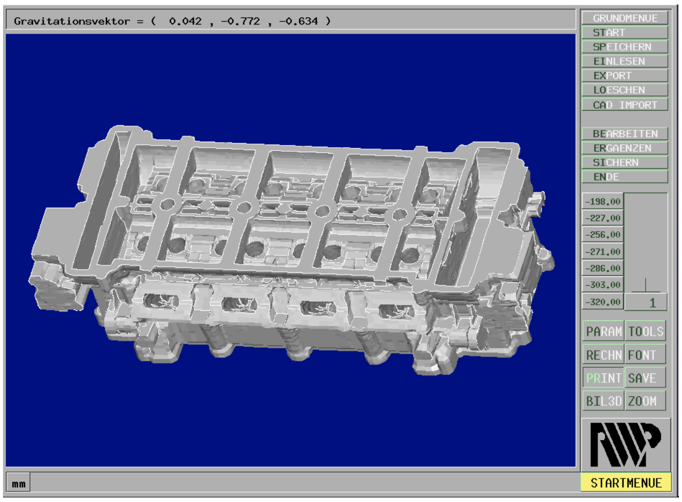

Figure 3:

A FEM enmeshment has a good volume and surface approximation even with a small number of elements

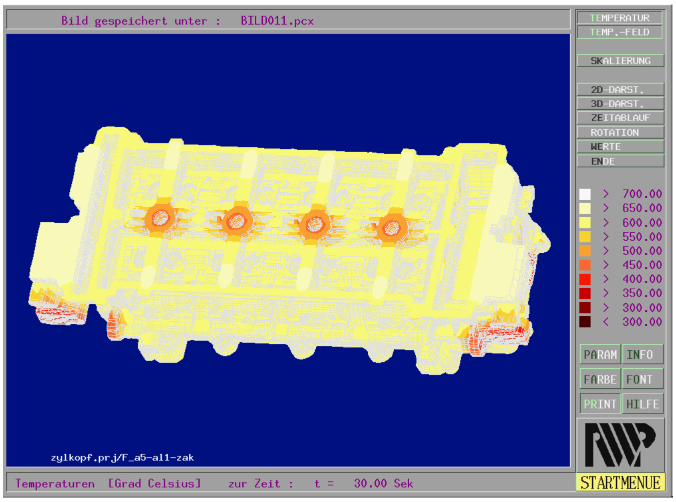

Figure 4:

With finite elements it is possible to get fast and precise results of various technical questions

It is obvious that the surface area has an important influence on the heat transfer.

They may have a nice appearance, but it is a disadvantage for a user that some programs don't show the grid. Anyone who is deeply involved with numerical simulation knows that the mesh has an influence on the results. But if the user doesn't see the mesh, he can't see possible flaws. This makes it difficult to interpret the results because it is not obvious whether problems are caused by the FDM/FVM mesh or by casting defects.

The disadvantage of the rectangular grid must be compensated for with a considerably finer mesh. While FEM calculations with their reasonable number of elements can be done on normal PCs, FDM/FVM programs tend to require expensive cluster computers with multiple processors to calculate complex problems with their large number of elements.

Occasionally you may hear, "I am not interested in details concerning hardware". Consider the enormously increasing processor speeds combined with decreasing prices on desktop PC's. Even today complex calculations are limited due to restrictions in the hardware options & availability.

The requirements for the accuracy of calculations are increasing at least as quickly as hardware capability. While it used to be sufficient to calculate simple solidification problems, future combined calculations of temperature fields, micro structure, residual stress and distortion will be expected.